The Schooling Model

Jörn-Steffen Pischke

LSE

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Setup

Individual maximises life-cycle utility in consumption

- Assumes fixed labor supply
- Perfect credit markets

Question: How much investment in earnings capacity should the individual undertake?

- Given the assumptions, consumption and investment decisions can be separated (Fisher separation)

Investment side

$$
\begin{aligned}
Y &= wH = wf(S, \cdot) \\
H &= \text{human capital } = f(S, \cdot) \\
S &= \text{number of years of schooling} \\
w &= \text{rental rate of a unit of human capital (a market price)}
\end{aligned}
$$

Maximise PV of earnings

$$
\max V = \int_S^T Ye^{-rt} dt,
$$

i.e. the only cost of schooling is foregone earnings.

Life time earnings

$$\begin{aligned} V &= \int_S^T Y e^{-rt} dt \\ &= wf(S, \cdot) \int_S^T e^{-rt} dt \\ &= \frac{w}{r} f(S, \cdot) \left[e^{-rS} - e^{-rT} \right] \end{aligned}$$

Simplification: $T \to \infty$

$$V = \frac{w}{r} f(S, \cdot) e^{-rS}$$

Optimisation

$$\max V = \frac{w}{r} f\left(S, \cdot\right) e^{-rS}$$

is equivalent to

$$\max \ln V = \ln w - \ln r + \ln f\left(S, \cdot\right) - rS$$

First order condition:

$$\frac{\partial \ln V}{\partial S} = \frac{f_S}{f} - r = 0 \Leftrightarrow f_S = rf$$

where

$$f_S = \frac{\partial f\left(S, \cdot\right)}{\partial S}$$

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$$\text{foregone earnings} = wf(S, \cdot)$$

- Marginal benefit of schooling:

$$\text{PV of increase in future earnings} = \frac{wf_S}{r}$$

The Mincer earnings function

How do we get from the FOC of the schooling problem to an earnings function like Mincer's, a regression of $\ln Y$ on S? Recall

$$\frac{\partial \ln f(S, \cdot)}{\partial S} = \frac{f_S}{f}$$

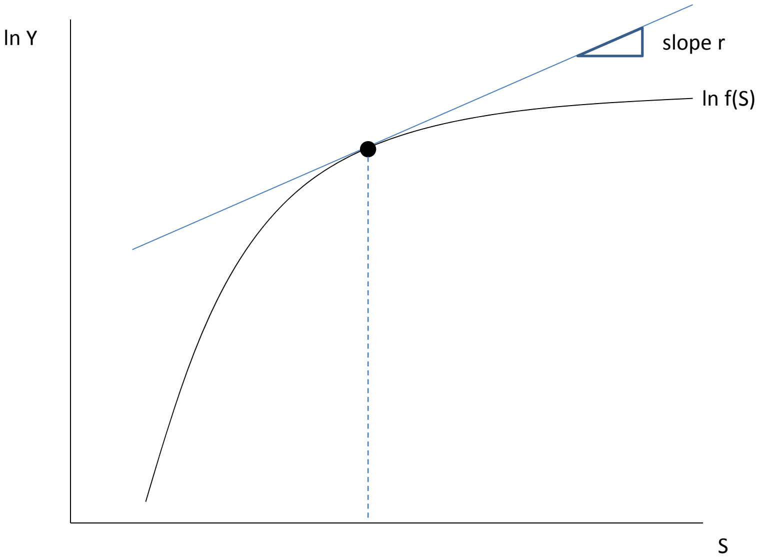
and from the FOC

$$\frac{\partial \ln f(S, \cdot)}{\partial S} = \frac{f_S}{f} = r$$

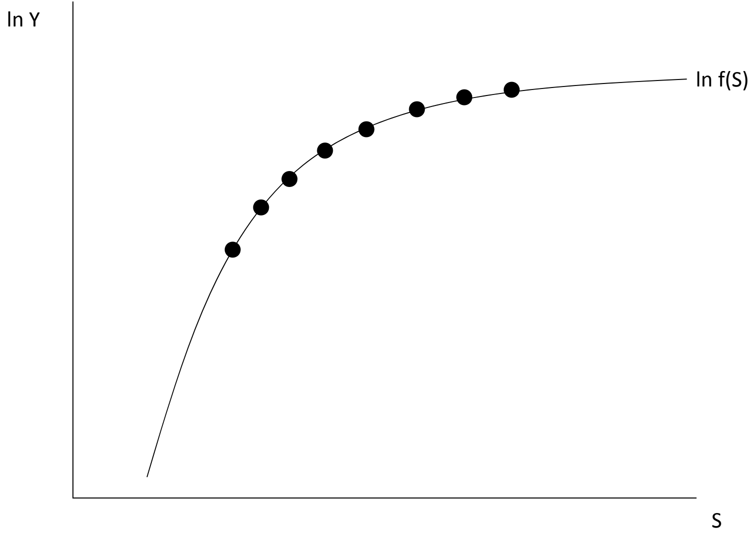
Integrate to get

$$\ln Y = \ln wf(S, \cdot) = const + rS$$

The FOC visually



Random choice of schooling



Why should the Mincer earnings function be linear?

The function $\ln f(S, \cdot)$ is only linear if $f(S, \cdot) = e^{aS}$. But there is no interior optimum if $r \neq a$.
Check the second order condition:

$$\text{FOC} \quad : \quad \frac{\partial \ln V}{\partial S} = \frac{f_S}{f} - r = 0$$

$$\text{SOC} \quad : \quad \frac{\partial^2 \ln V}{\partial S^2} = \frac{f_{SS} f - f_S^2}{f^2} = \frac{f_{SS}}{f} - \frac{f_S^2}{f^2} = \frac{f_{SS}}{f} - \frac{r f_S}{f} < 0$$

$$f_{SS} - r f_S < 0$$

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- The FOC is

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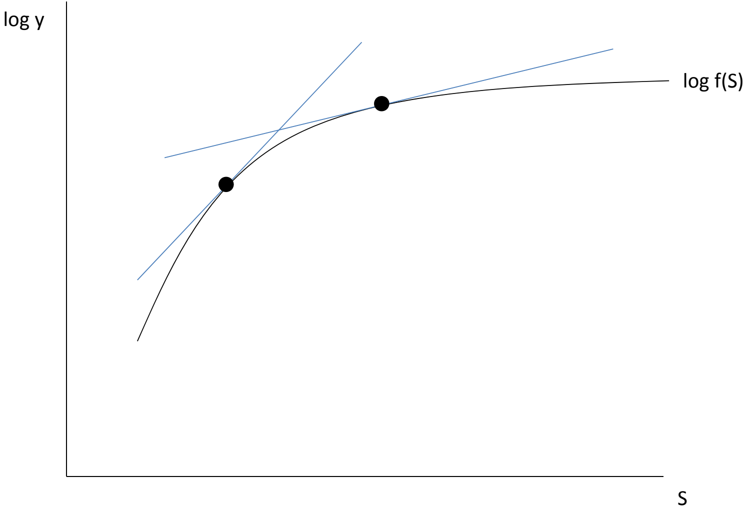
- Start by totally differentiating the FOC:

$$f_{SS}\, dS = r f_S\, dS + f dr$$

Collect terms

$$\begin{aligned} (f_{SS} - r f_S)\, dS &= f dr \\ \frac{dS}{dr} &= \frac{f}{f_{SS} - r f_S} < 0 \end{aligned}$$

Comparative statics visually



Ability

Let

$$H = f(S, A)$$

Totally differentiate the FOC

$$\begin{aligned} f_S(S, A) &= rf(S, A) \\ f_{SS}dS + f_{SA}dA &= rf_S dS + rf_A dA \\ (f_{SS} - rf_S)\, dS &= -(f_{SA} - rf_A)\, dA \\ \frac{dS}{dA} &= -\frac{\overbrace{f_{SA}}^{?} - \overbrace{rf_A}^{(+)}}{\underbrace{f_{SS} - rf_S}_{(-)\ (\text{SOC})}} \end{aligned}$$

The relationship between ability and schooling

$$\frac{dS}{dA} = -\frac{\overbrace{f_{SA}}^{?} - \overbrace{rf_A}^{(+)}}{\underbrace{f_{SS} - rf_S}_{(-)\ (\text{SOC})}}$$

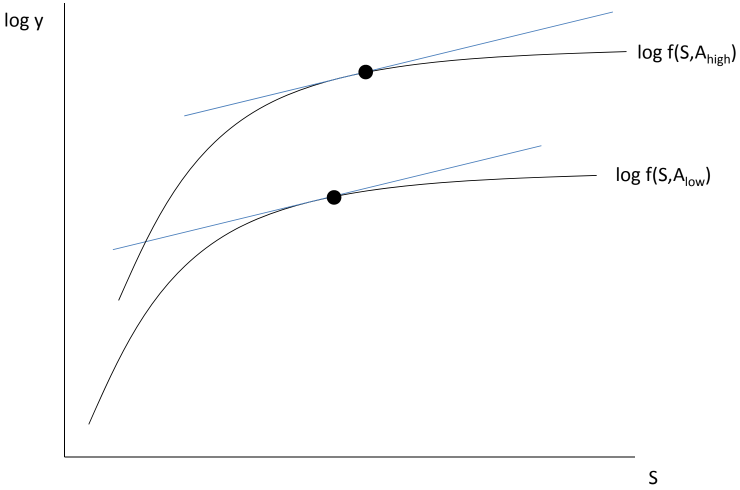
So $dS/dA > 0$ only if f_{SA} is big enough.
What is rf_A?

- The cost of a year of schooling is higher for the more able. Everybody gives up wf for a year in school, but $f_A > 0$, so that's bigger for the more able.
- Why is ability different from the wage?
  - A only matters if it doesn't enter proportionately. Make

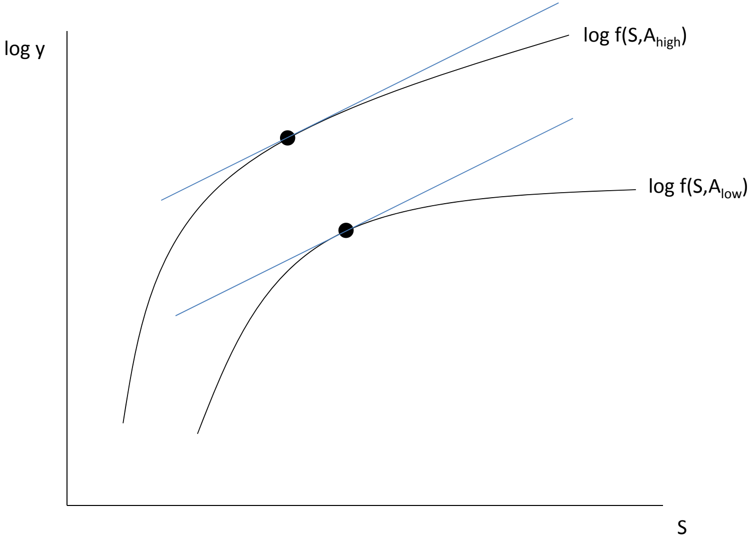
$$f(S, A) = A\delta(S)$$

$$f_{SA} = \delta' = \frac{A\delta'\delta}{A\delta} = \frac{f_S f_A}{f} = rf_A$$

Ability and schooling are complements



Ability and schooling are substitutes



Back to the Mincer earnings function

We have considered

- Variance introduced by interest rate variation
- Random variation in schooling
- Variance introduced by ability

None of these lead to a linear relationship between $\ln Y$ and S in the data.